TE-ESN: Time Encoding Echo State Network for Prediction Based on Irregularly Sampled Time Series Data

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ABSTRACT

Prediction based on Irregularly Sampled Time Series (ISTS) is of wide concern in real-world applications. Different from the ordinary time series, ISTS is characterized by irregular time intervals of intraseries and different sampling rates of inter-series. However, existing methods have suboptimal predictions due to artificially introducing new dependencies in a time series and biasedly learning relations among time series when modeling these two characteristics. In this work, we propose a novel Time Encoding (TE) mechanism. TE can embed the time information as time vectors in the complex domain. It has the properties of absolute distance and relative distance under different sampling rates, which helps to represent two irregularities. Meanwhile, we create a new model named Time Encoding Echo State Network (TE-ESN). It is the first ESNs-based model that can process ISTS data. Experiments on one chaos system and three realworld datasets show that TE-ESN performs better than all baselines and has better reservoir property.

CCS CONCEPTS

- Computing methodologies \rightarrow Neural networks.

KEYWORDS

irregularly sampled time series, echo state networks, time encoding.

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1 INTRODUCTION

Time series prediction widely exists in many scenarios, such as healthcare and meteorology [20, 21]. However, in real-world applications, it's usually Irregularly Sampled Time Series (ISTS), having two irregularities under the aspects of intra-series and inter-series:

• Intra-series irregularity is the irregular time intervals between observations within a time series. In Figure 1, the time between a

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COVID-19 patient's blood sample could be 1 hour or even 7 days. Uneven intervals changes the dependency between observations and large intervals add time sparsity factor [11].

• Inter-series irregularity is the different sampling rates among time series. For a COVID-19 patient in Figure 1, heart rate is measured in seconds, while blood sample is collected in days. The difference of sampling rates is not conducive to data preprocessing and model design [12].

However, grasping both two irregularities is challenging. In realworld applications, a model usually has multiple time series as input. If seeing the input as a multivariate time series, data alignment with up/down-sampling and imputation occur. But it will artificially introduce some new dependencies while omit some original dependencies, causing suboptimal prediction [16]; If seeing the input as multiple separated time series and modeling them separately, it will encounter the problem of bias, embedding stronger dependency in high sampled time series due to smaller time intervals. This is not necessarily the case, for example, although the detection of blood pressure is not frequent than heart rate in clinical practice, its values have a strong diurnal correlation [18].

In order to get rid of the above dilemmas, modeling all irregularities without introducing new dependency is feasible [8]. The premise is that ISTS can't be interpolated, which makes the alignment impossible, leading to batch gradient descent for multivariate time series hard to implement, aggravating the non-converging and instability of error Back Propagation Recurrent Neural Networks (BPRNNs) [5], the basis of existing methods [16]. Echo State Networks (ESNs) is a simple type of RNNs and can avoid nonconverging and computationally expensive by applying least square problem as the alternative training method [9]. But ESNs can only process uniform TS by assuming time intervals are equally distributed, with no mechanism to model ISTS. For solving all the difficulties mentioned above, we design a new structure to enable ESNs to handle ISTS data, where a novel mechanism makes up for the disadvantage of no learning of irregularity.

- We introduce a novel mechanism named Time Encoding (TE). TE represents time points as dense vectors in complex domain. It injects the absolute and relative distance properties to model both intra-series irregularity and inter-series irregularity of ISTS.
- We design a model named Time Encoding Encoding Echo State Network (TE-ESN). In addition to the ability of modeling both two ISTS irregularities, TE-ESN can learn the long short-term memories longitudinally and fuses the relations horizontally.
- We evaluate TE-ESN for early prediction and one-step-ahead forecasting on four datasets. TE-ESN outperforms state-of-the-art models and has better reservoir property.

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Figure 1: An ISTS example of a COVID-19 patient and the structure of TE-ESN

2 RELATED WORK

Existing methods can be divided into two categories:

Missing data perspective. It discretizes the time axis into nonoverlapping intervals, points without data are considered as missing data. M-RNN [11] handled missing data by operating time series forward and backward. GRU-D [3] used decay rate to weigh the correlation between missing data and other data. But data imputation may artificially introduce new dependency beyond original relations and totally ignore ISTS irregularities.

Raw data perspective. It constructs models which can directly receive ISTS as input. T-LSTM [1] used the elapsed time function for modeling irregular time intervals. IPN [15] used three time perspectives for modeling different sampling rates. However, they just performed well in the univariate time series, for multiple time series, they had to apply alignment first, causing the data missing in some time points, back to the defects of the first category.

3 TIME ENCODING ECHO STATE NETWORK

3.1 Definitions

DEFINITION 1 (IRREGULARLY SAMPLED TIME SERIES ISTS). A time series u with sampling rate $r_s(d), d \in \{1, ..., D\}$ has several observations distributed with time $t, t \in \{1, ..., T\}$. u_t^d represents an observation of a time series with sampling rate $r_s(d)$ in time t.

ISTS has two irregularities: (1) Irregular time intervals of intraseries: $t_i - t_{i-1} \neq t_j - t_{j-1}$. (2) Different sampling rate of inter-series: $r_s(d_i) \neq r_s(d_j)$. For prediction tasks, one-step-ahead forecasting is using the observed data $u_{1:t}$ to predict the value of u_{t+1} , and continues over time; Early prediction is using the observed data $u_{1:t}$ ($t < t_{pre}$) to predict the classes or values in time t_{pre} .

DEFINITION 2 (TIME ENCODING TE). Time encoding mechanism aims to design methods to embed and represent every time point information of a time line.

TE mechanism extends the idea of Positional Encoding (PE) [7, 17] in natural language processing, shown in Equation 1. Where *pos* indicates the position of a word, d_{model} is the embedding dimension.

$$\begin{cases} PE(pos, 2i) = \sin\left(\frac{pos}{10000^{\frac{2i}{d_{model}}}}\right) \\ PE(pos, 2i+1) = \cos\left(\frac{pos}{10000^{\frac{2i}{d_{model}}}}\right) \end{cases}$$
(1)

3.2 Time Encoding Mechanism

3.2.1 Time Vector with Fixed Sampling Rate. Only consider one time series, whose irregularity is just reflected in the irregular

time intervals, we apply Time Vector (TV) to note the time as vector. Each time vector has d_{TV} embedding dimensions. Each dimension corresponds to a sinusoidal wave, forming a geometric progression from 2π to $MT\pi$. MT is the maximum number of input time points.

$$TV(t) = [..., \sin(c_i t), \cos(c_i t), ...]$$

$$c_i = MT^{-\frac{2i}{d_{TV}}}, i = 0, ..., \frac{d_{TV}}{2} - 1$$
(2)

TV can simulate the time intervals between two observations by its properties of absolute distance and relative distance.

PROPERTY 1 (ABSOLUTE DISTANCE PROPERTY). For two time points with distance p, the time vector in time point t + p is the linear combination of the time vector in time point t.

$$TV(t+p) = (a,b) \cdot TV(t)$$

$$a = TV(p,2i+1), b = TV(p,2i)$$
(3)

PROPERTY 2 (RELATIVE DISTANCE PROPERTY). The product of time vectors of two time points t and t + p is negatively correlated with their distance p.

$$TV(t) \cdot TV(t+p) = \sum_{i=0}^{\frac{d_{TV}}{2}-1} \cos(c_i p)$$
(4)

3.2.2 Time Encoding with Different Sampling Rates. When the input is multi-series, the irregularity of different sampling rates occurs. Then, TV will encounter the problem of bias, embedding more associations between observations with high sampling rate according to the Property 2. Thus, we design an advanced version of time vector, noted Time Encoding (TE), to encode time within multiple ISTS. TE extends TV to complex-valued domain. For a time point *t* in the *d*-th ISTS with $r_s(d)$ sampling rate, the time code is in Equation 5, where ω is the frequency.

$$TE(d,t) = e^{i(\omega t)}, \omega = \omega_d \cdot r_s^{-1}(d)$$
(5)

TE not only keeps the property 1 and 2 but also incorporates the influence of frequency ω , making time codes consistent at different sampling rates. ω reflects the sensitivity of observation to time, where a large ω leads to more frequent changes of time codes and more difference between the adjacent time points.

PROPERTY 3 (RELATIVE DISTANCE PROPERTY WITH ω). The product of time encoding of two time points t and t+p is positive correlated with the frequency ω .

$$TE(t) \cdot TE(t+p) = e^{i\omega t} \cdot e^{i\omega(t+p)} = e^{i\omega(2t+p)}$$
(6)

In TE, we set $\omega = \omega_d \cdot r_s^{-1}(d)$. ω_d is the frequency parameter of *d*-th sampling rate. TE fuses the sampling rate term $r_s^{-1}(d)$ to avoid the bias of time vector causing by only considering the distance *p*. Each time point can be embedded into d_{TE} dimensions with more options of frequencies by setting different $\omega_{j,k}$ in Equation 7. $\omega_{j,k}$ means the time vector in dimension *j* has *k* frequencies.

$$TE(d,t) = e^{i(\omega t)}, \omega = \omega_{j,k} \cdot r_s^{-1}(d)$$

$$j = 0, ..., d_{TV} - 1, k = 0, ..., K - 1$$
(7)

3.2.3 The Relations between Different Mechanisms.

- TV is a special case of TE. If we set $\omega_{k,j} = c_i$, then TE(d, t) = TV(t, 2i + 1) + iTV(t, 2i).
- TE is a special case of r · e^{i · (ωx+θ)}. θ = 0 as we focus more on the relation between different time points than the value of the first point; r is the representation of observations and leave it to learn by computing models. Besides, TE inherits the properties of position-free offset transformation and boundedness [19].

3.3 Injecting Time Encoding Mechanism into Echo State Network

ESN is a fast and efficient RNN. A typical ESN consists of an input layer $W_{in} \in \mathbb{R}^{N \times D}$, a reservoir layer $W_{res} \in \mathbb{R}^{N \times N}$ and an output layer $W_{out} \in \mathbb{R}^{M \times N}$. $u(t) \in \mathbb{R}^D$, $x(t) \in \mathbb{R}^N$ and $y(t) \in \mathbb{R}^M$ denote the input value, reservoir state and output value at time *t*. The existing ESNs-based methods cannot model the irregularities of ISTS. Thus, we make up for this by proposing Time Encoding Echo State Network (TE-ESN) as shown in Figure 1.

3.3.1 Time Encoding Phase. TE-ESN has *D* reservoirs, assigning each input time series an independent reservoir by:

$$\tilde{x}_{t}^{d} = \gamma_{f} x_{t}^{d'} + (1 - \gamma_{f}) x^{D \setminus d} \quad \text{Reservoir}$$

$$x_{t}^{d'} = \gamma_{I} x_{t}^{d} + (1 - \gamma_{I}) (x_{t-1}^{d} + x_{t-k}^{d}) \quad \text{Long short}$$

$$x_{t}^{d} = \tanh(TE(d, t) + W_{in}^{d} u_{t}^{d} + W_{res}^{d} \tilde{x}_{t-1}^{d}) \quad \text{Time encoding} \qquad (8)$$

 $x^{D\setminus d} = \frac{1}{D-1} \sum_{i \in D \setminus d} \tilde{x}^i \quad Neighbor$

- Time encoding mechanism (TE). TE-ESN integrates time by changing reservoir state through TE term to *Time encoding state*.
- Long short-term memory mechanism (LS). TE-ESN incorporates long short term memories from former *k* time to *Long short state*.
- Series fusion (SF). TE-ESN considers the horizontal time series to change *Reservoir state* by the *Neighbor state* in other time series.

3.3.2 Time Decoding Phase. For final value prediction, TE-ESN decodes the time information and get the real estimated value at time t_{pre} by Equation 9. Further, by changing the time t_{pre} , we can get different prediction results in different time points.

$$y(t_{pre}) = W_{out}(\tilde{x}(t) - TE(t_{pre}))$$
(9)

$$\begin{split} \min_{W_{out}} ||Y_{pre} - Y||_2^2 + \lambda ||W_{out}||_2^2 \\ W_{out} &= Y(\tilde{X} - TE)^T ((\tilde{X} - TE)(\tilde{X} - TE)^T + \lambda I)^{-1} \end{split} \tag{10}$$

Algorithm 1 shows how TE-ESN predicts. Assuming reservoir size is *N*, maximum time is *T*, input has *D* data, the complexity is:

$$C = O(\alpha T N^2 + T N D) \tag{11}$$

Algorithm 1 TE-ESN

```
Input: training input X_{train} = \{x_t^d, t\}, teacher signal Y_{train} = \{y_t^d, t\},
    test input X_{test} = \{x_t^d, t\},maximum time MT,
     leaky rate \gamma_l, fusion rate \gamma_f, time span k, regularization \lambda
    input scale w_{in}, spectral radius \rho(W_{res}), sparsity \alpha,
Output: Ypre: prediction result.
 1: Randomly initialized W_{in} in [-w^{in}, w^{in}];
 2: Randomly initialized W_{res} with \alpha and \rho(W_{res}).
 3: for i = 1 to |U_{train}| do
        for t = 1 to MT do
 4:
            Compute TE(d, t) by Equation 7
 5:
 6:
            Compute \tilde{x}(t) by Equation 8
 7:
        end for
 8: end for
 9: \tilde{X} = {\tilde{x}(t)}
10: TE = \{TE(t)\}
11: Compute W_{out} by Equation 10
12: for t = 1 to T_{test} do
        Compute TE_{test}(d, t) by Equation 7
13:
14:
        Compute \tilde{x}_{test}(t) by Equation 8
15: end for
16: \tilde{X}_{test} = {\tilde{x}_{test}(t)}
17: TE_{test} = \{TE_{test}(t)\}
18: Y_{pre} = W_{out}(\tilde{X}_{test} - TE_{test})
```

4 EXPERIMENTS

4.1 Experiment Setting

Datasets

- *MG* [13] chaotic system. $y(t+1) = y(t) + \delta(a \frac{y(t-\frac{\tau}{\delta})}{1+y(t-\frac{\tau}{\delta})^n} by(t))$. $\delta, a, b, n, \tau, y(0) = 0.1, 0.2, -0.1, 10, 17, 1.2. t$ random increases.
- *SILSO* [2] provides open-source monthly sunspot series from 1749 to 2020. It has irregular time intervals, from 1 to 6 month.
- USHCN [14] has U.S. daily climate data from 1887 to 2014. Time intervals from 1 to 7 days. Sampling rates from 0.33 to 1 per day.
- COVID-19 [23] contains blood samples with 80 features and 6877 records, from Tongji Hospital, Wuhan, China. Time intervals are from 1 minus to 12 days. Sampling rates are from 0 to 6 per day.

Baselines

- *BPRNNs-based*: There are 3 methods designed for ISTS data with BP training M-RNN [11], T-LSTM [1] and GRU-D [3].
- ESNs-based: There are 4 methods designed based on ESNs ESN [9], Leaky-ESN [10], DeepESN [6] and LS-ESN [24].

Metrics Use genetic algorithm [25] to optimize hyper-parameters in Table 5. Evaluate prediction by AUC-ROC and MSE. Evaluate network property by Memory Capability (MC) [4]: $MC = \sum_{k=0}^{\infty} r^2(u(t-k), y(t))$

4.2 Results

Prediction Results. TE-ESN outperforms all baselines on four datasets shown in Table 1 and Figure 2. TE-ESN is better than TV-ESN in multivariable time series (COVID-19, USHCN) shows the effect of Property 3; TE-ESN is better than TV-ESN in univariable time series (SILSO, MG) shows the advantage of multiple frequencies options of TE.

Time Encoding Mechanism Analysis. Dot product between two sinusoidal positional encoding decreases with increment of absolute value of distance [22]. Figure 4 and Table 3 show that using multiple frequencies will enhance monotonous of negative correlation and can improve the accuracy.



Figure 2: Lactic dehydrogenase (LDH) forecasting for a 70-year-old female COVID-19 patient

	BPRNNs-based				ESNs-based				Ours	
	M-RNN	T-LSTM	GRU-D	ESN	leaky-ESN	DeepESN	LS-ESN	TV-ESN	TE-ESN	
MG	0.232±0.005	0.216 ± 0.003	0.223±0.005	0.229 ± 0.001	0.213 ± 0.001	0.197±0.000	0.198 ± 0.000	0.204 ± 0.001	0.195±0.001	
SILSO	2.950 ± 0.740	2.930 ± 0.810	2.990 ± 0.690	3.070 ± 0.630	2.950 ± 0.590	2.800±0.730	2.540 ± 0.690	2.540 ± 0.790	2.390 ± 0.780	
USHCN	0.752±0.320	0.746 ± 0.330	0.747 ± 0.250	0.868 ± 0.290	0.857 ± 0.200	0.643±0.120	0.663 ± 0.150	0.647 ± 0.150	0.640 ± 0.190	
COVID 10	0.098 ± 0.005	0.096 ± 0.007	0.100 ± 0.005	0.136 ± 0.006	0.135 ± 0.007	0.129 ± 0.006	0.120 ± 0.007	0.115 ± 0.005	0.093±0.005	
COVID-19	$0.959 {\pm} 0.004$	0.963±0.003	$0.963 {\pm} 0.004$	$0.941 {\pm} 0.003$	$0.942 {\pm} 0.003$	$0.948 {\pm} 0.003$	$0.949 {\pm} 0.003$	$0.958 {\pm} 0.002$	0.965 ± 0.002	

Table 1: Prediction results of nine methods on four datasets (COVID-19 mortality in AUC-ROC; Others in MSE)

Parameters	Value range	Parameters	Value range
$w^{in}, lpha, ho$ k	(0,1] {2,4,6,8,10,12}	γι, γ _f λ	$ \begin{bmatrix} 0,1 \\ \{10^{-4},10^{-2},1 \} \\$

Table 2: Search settings of hyper-parameters

$c_{i}, 32$	$c_{i}, 64$	$\omega_{d,i}, 32$	$\omega_{d,i}, 64$
0.226 ± 0.001	0.204 ± 0.001	0.210 ± 0.001	0.193±0.001
2.690 ± 0.600	2.540±0.79	2.550 ± 0.750	2.390 ± 0.780
0.681 ± 0.180	0.670 ± 0.200	0.673 ± 0.170	0.640 ± 0.190
$0.105 {\pm} 0.006$	0.099 ± 0.005	$0.101 {\pm} 0.005$	0.093±0.005
$0.949 {\pm} 0.002$	0.952 ± 0.003	$0.950 {\pm} 0.002$	0.965 ± 0.002
	$\begin{array}{c} c_i, 32 \\ 0.226 \pm 0.001 \\ 2.690 \pm 0.600 \\ 0.681 \pm 0.180 \\ 0.105 \pm 0.006 \\ 0.949 \pm 0.002 \end{array}$	$\begin{array}{cccc} c_i, 32 & c_i, 64 \\ \hline 0.226\pm0.001 & 0.20\pm0.001 \\ 2.690\pm0.600 & 2.540\pm0.79 \\ 0.681\pm0.180 & 0.670\pm0.200 \\ 0.105\pm0.006 & 0.099\pm0.005 \\ 0.949\pm0.002 & 0.952\pm0.003 \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

Table 3: Prediction results of TE-ESN with different ω , d_{TE}

	w/o TE	w/o LS	w/o SF	TE-ESN
MG	0.210±0.001	0.213±0.001	0.193±0.001	0.193±0.001
SILSO	2.790 ± 0.630	2.930 ± 0.690	2.390 ± 0.780	2.390±0.780
USHCN	0.713 ± 0.120	0.757 ± 0.210	0.693 ± 0.160	0.640±0.190
COVID-19	0.135 ± 0.006	0.130 ± 0.006	0.125 ± 0.007	0.093±0.005
	0.943 ± 0.003	0.949 ± 0.003	0.956 ± 0.003	0.965±0.002

Table 4: Prediction results with different mechanisms

	w^{in}	α	ρ	γı	k	Υf	λ
MG	1	0.1	0.7	0.8	6	1.0	10^{-2}
SILSO	1	0.1	0.6	0.8	10	1.0	10^{-2}
USHCN	1	0.1	0.7	0.8	12	0.8	10^{-2}
COVID 10	1	0.2	0.8	0.8	2	0.8	10^{-2}
COVID-19	1	0.3	0.9	0.7	4	0.9	10^{-2}

Table 5: Best settings of hyper-parameters of TE-ESN

	ESN	leaky-ESN	DeepESN	LS-ESN	w/o TE	TE-ESN
M	35.05	39.65	42.98	46.05	40.46	47.83

Table 6: Memory capacity results of ESNs-based methods

TE-ESN Ablation Study of TE, LS and SF. Table 4 show that all theses three mechanisms contribute to the final prediction tasks. Hyper-Parameters Analysis of TE-ESN. Figure 3 shows that setting different hyper-parameters for each reservoir has little effect on the prediction results. Table 5 is the best settings.



Figure 3: Prediction of TE-ESN with different ρ and k



Figure 4: Dimension and frequency setting of time encoding

Memory Capability Analysis. Table 6 shows that TE-ESN obtains the best MC value and TE can increase the memory capability.

5 **CONCLUSIONS**

In this paper, we propose a novel Time Encoding (TE) mechanism in complex domain to model the time information of ISTS. It can represent the irregularities of intra-series and inter-series. We create a novel Time Encoding Echo State Network (TE-ESN), which is the first method to enable ESNs to handle ISTS. We evaluate the method and give several model related analysis in two prediction tasks on four datasets. The results show that TE-ESN outperforms the existing state-of-the-art and has good properties. Future works will focus on the dynamic reservoir properties and hyper-parameters optimization of TE-ESN, and will incorporate deep structures to TE-ESN for better prediction accuracy.

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